Introduction to chaos
Sarah Keeley
(giving lecture material prepared by Tim Palmer)
Vilhelm Bjerknes (1862-1951)

Proposed weather forecasting as a deterministic initial value problem based on the laws of physics.
Lewis Fry Richardson (1881-1953)

The first numerical weather forecast
“Why have meteorologists such difficulty in predicting the weather with any certainty? Why is it that showers and even storms seem to come by chance ... a tenth of a degree (C) more or less at any given point, and the cyclone will burst here and not there, and extend its ravages over districts that it would otherwise have spared. If (the meteorologists) had been aware of this tenth of a degree, they could have known (about the cyclone) beforehand, but the observations were neither sufficiently comprehensive nor sufficiently precise, and that is the reason why it all seems due to the intervention of chance”

Poincaré, 1909
Edward Lorenz (1917 –2008)

“… one flap of a sea-gull’s wing may forever change the future course of the weather” (Lorenz, 1963)

\[
\begin{align*}
\dot{X} &= -\sigma X + \sigma Y \\
\dot{Y} &= -XZ + rX - Y \\
\dot{Z} &= XY - bZ
\end{align*}
\]
The Lorenz (1963) attractor, the prototype chaotic model.....
\[ \frac{dX}{dt} = F[X] \] is a nonlinear system

\[ \Rightarrow \frac{d\delta X}{dt} = \frac{dF}{dX} \delta X \equiv J \delta X \]

Since \( F \) is a nonlinear function of \( X \)

\[ \Rightarrow J = J(X) \]
Lothar: 08Z, 26 Dec. 1999

- 100 Fatalities
- 400 million trees blown down
- 3.5 million electricity users affected for 20 days
- 3 million people without water
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Lothar (T+42 hours)

Ensemble forecast of the French / German storms (surface pressure)
Start date 24 December 1999 : Forecast time T+42 hours
Thanks to Rob Hine
Charlie is planning to lay concrete tomorrow.

Should he?

Let $p$ denote the probability of frost

Charlie loses $L$ if concrete freezes.

- But Charlie has fixed (eg staff) costs
- There may be a penalty for late completion of this job.
- By delaying completion of this job, he will miss out on other jobs.
- These cost $C$

Is $Lp > C$? If $p > C/L$ don’t lay concrete!
Introduction to chaos for:
Seasonal climate prediction

Atmospheric predictability arises from slow variations in lower-boundary forcing
Edward Lorenz (1917 – 2008)

\begin{align*}
\dot{X} &= -\sigma X + \sigma Y + f \\
\dot{Y} &= -XZ + rX - Y + f \\
\dot{Z} &= XY - bZ
\end{align*}

What is the impact of $f$ on the attractor?
Add external steady forcing $f$ to the Lorenz (1963) equations.

The influence of $f$ on the state vector probability function is itself predictable.
Seasonal Probability Forecasts (ECMWF / HOPE coupled model)
Winter 1997/98 from October 1997

Blue: More likely to be wetter than normal, Red: More likely to be drier than normal
The tropical Pacific ocean/atmosphere
Introduction to chaos for:
Stochastic parametrisation
Eg 2) Lorenz(1963) in an EOF

\[ \dot{a}_1 = 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3 \]
\[ \dot{a}_2 = -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3 \]
\[ \dot{a}_3 = -0.63a_1 - 13a_3 + 0.43a_1a_2 + 0.49a_2a_3 \]

3\(^{rd}\) EOF only explains 4\% of variance (Selten, 1995).

Parametrise it?
Lorenz(1963) in a truncated EOF basis with parametrisation of $a_3$

\[
\dot{a}_1 = 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3
\]
\[
\dot{a}_2 = -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3
\]
\[
a_3 = f(a_1, a_2)
\]

Good as a short-range forecast model (using L63 as truth), but exhibits major systematic errors compared with L63, as, by Poincaré-Bendixon theorem, the system cannot exhibit chaotic variability – system collapses onto a point attractor.
Stochastic-Lorenz(1963) in a truncated EOF basis

\[ \dot{a}_1 = 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3 \]
\[ \dot{a}_2 = -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3 \]
\[ a_3 = \beta \]

Stochastic noise
Lorenz attractor

Truncated Stochastic-Lorenz attractor – weak noise

Truncated Stochastic-Lorenz attractor

Error in mean and variance

Palmer, 2001 (acknowledgment to Frank Selten)