Basics of Coupled Assimilation

Difficulties in Coupled Assimilation

Potential benefits of coupled assimilation

Coupled assimilation at ECMWF
Suppose we model the temperature in the room, but we split the room in half and have one temperature for each half; $x_1$ and $x_2$.

Now let us measure the temperature in each half of the room; $y_1$ and $y_2$. 
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Uncoupled assimilation (3DVar)
A simple example 1

Suppose we model the temperature in the room, but we split the room in half and have one temperature for each half; $x_1$ and $x_2$. Now let us measure the temperature in each half of the room; $y_1$ and $y_2$.

**Uncoupled assimilation (3DVar)**

\[
x = x_1, \quad y = y_1, \quad H = 1, \quad R = \sigma_y^2, \quad P_b = \sigma_x^2
\]

\[
J_1(x) = (x_{b_1} - x_1)\sigma_x^{-2}(x_{b_1} - x_1) + (y_1 - x_1)\sigma_y^{-2}(y_1 - x_1)
\]
A simple example 1

Suppose we model the temperature in the room, but we split the room in half and have one temperature for each half; $x_1$ and $x_2$.
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Uncoupled assimilation (3DVar)

$$x = x_1, y = y_1, H = 1, R = \sigma_{y_1}^2, P_b = \sigma_{x_1}^2$$

$$J_1(x) = (x_{b_1} - x_1)\sigma_{x_1}^{-2}(x_{b_1} - x_1) + (y_1 - x_1)\sigma_{y_1}^{-2}(y_1 - x_1)$$

$$x = x_2, y = y_2, H = 1, R = \sigma_{y_2}^2, P_b = \sigma_{x_2}^2$$

$$J_2(x) = (x_{b_2} - x_2)\sigma_{x_2}^{-2}(x_{b_2} - x_2) + (y_2 - x_2)\sigma_{y_2}^{-2}(y_2 - x_2)$$
A simple example 2

**Coupled assimilation (3DVar)**

\[
x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad H = I, \quad R = \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}, \quad P_b = \begin{bmatrix} \sigma_x^2 & \sigma_{x12}^2 \\ \sigma_{x12}^2 & \sigma_{x2}^2 \end{bmatrix}
\]

\[
J(x) = \begin{bmatrix} x_{b1} - x_1 \\ x_{b2} - x_2 \end{bmatrix}^T \begin{bmatrix} \sigma_x^2 & \sigma_{x12}^2 \\ \sigma_{x12}^2 & \sigma_x^2 \end{bmatrix}^{-1} \begin{bmatrix} x_{b1} - x_1 \\ x_{b2} - x_2 \end{bmatrix} + (y_1 - x_1)\sigma_{y1}^{-2}(y_1 - x_1) + (y_2 - x_2)\sigma_{y2}^{-2}(y_2 - x_2)
\]
Suppose we stop observing \( y_2 \).

- In uncoupled 3DVar, \( x_2 = x_{b2} \), i.e. nothing happens for this variable.
- In coupled 3DVar \( x_2 \) is still updated if \( \sigma^2_{x_{12}} \neq 0 \).
Suppose we stop observing $y_2$.

- In uncoupled 3DVar, $x_2 = x_{b2}$, i.e. nothing happens for this variable.
- In coupled 3DVar $x_2$ is still updated if $\sigma^2_{x_{12}} \neq 0$.

If we never observe $y_2$, the *cross-covariance* $\sigma^2_{x_{12}}$ allows us to constrain $x_2$. 
Recall the 3DVar cost function:

\[ J(x) = \frac{1}{2}(x_b - x)^T P_b^{-1} (x_b - x) + \frac{1}{2} (y - \mathcal{H}(x))^T R^{-1} (y - \mathcal{H}(x)) \]

and its gradient

\[ -\nabla J(x) = P_b^{-1} (x_b - x) + H^T R^{-1} (y - \mathcal{H}(x)) \]

There are two ways \( x_2 \) can influence \( x_1 \):

- \( H \) is a function of both \( x_1 \) and \( x_2 \)
- \( P_{b12} \neq 0 \)
Recall the 4DVar cost function:

\[ J(x) = \frac{1}{2} (x_b - x)^T P_b^{-1} (x_b - x) + \frac{1}{2} \sum_k (y_k - G_k(x))^T R_k^{-1} (y_k - G_k(x)) \]

and its gradient

\[ -\nabla J(x) = P_b^{-1} (x_b - x) + \sum_k M_k^T H_k^T R_k^{-1} (y_k - G_k(x)) \]

There is a third way that \( x_2 \) can influence \( x_1 \):

- \( G_k \) is a function of both \( x_1 \) and \( x_2 \), i.e. the coupled model has mixed the information over time

Thus in 4DVar, the implied cross-covariance between elements of the system also allow for information to be transferred, on top of those supplied in \( P_b \).
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Coupled assimilation at ECMWF
Long assimilation windows

- The longer the assimilation window, the more observations we get to put into our systems.
- The longer the assimilation window, the more flow dependence we obtain in our solution - i.e. we become less reliant on the background error covariance that we specify at $t = 0$. 
Timescales in the Earth System

- Microscale turbulence: minutes
- Mesoscale storms (tornadoes/thunderstorms): hours
- Synoptic scale cyclones: days
- Planetary waves/blocking structures: weeks
- Intraseasonal features: months
- Seasonal cycles/ENSO: years
Timescales in the Earth System

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- Seasonal cycles/ENSO: years
- Internal waves: hours
- Tides: days
- Mesoscale eddies: weeks/months
- ENSO: years
- Thermohaline circulation: centuries
Tangent linear model and approximation
Strongly and weakly coupled assimilation

Solving the system as described previously is known as strongly coupled 4DVar.

- $P_b$ may or may not have off-diagonal blocks, i.e. cross-covariances
- Requires $M^T$ of the fully coupled system.

! Problem: what if the coupled system is implemented in entirely different computer codes?
Strongly and weakly coupled assimilation

Solving the system as described previously is known as strongly coupled 4DVar.

- $P_b$ may or may not have off-diagonal blocks, i.e. cross-covariances
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! Problem: what if the coupled system is implemented in entirely different computer codes?

One approach is known as weakly coupled 4DVar in which

\[
M = \begin{bmatrix} M_1 & M_{12} \\ M_{21} & M_2 \end{bmatrix} := \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \quad \text{i.e.} \quad M\mathbf{x} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} M_1 x_1 \\ M_2 x_2 \end{bmatrix}
\]

\[
M^T = \begin{bmatrix} M_1^T & M_{12}^T \\ M_{21}^T & M_2^T \end{bmatrix} := \begin{bmatrix} M_1^T & 0 \\ 0 & M_2^T \end{bmatrix} \quad \text{i.e.} \quad M^T\mathbf{x} = \begin{bmatrix} M_1^T & 0 \\ 0 & M_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} M_1^T x_1 \\ M_2^T x_2 \end{bmatrix}
\]
Weakly coupled assimilation in incremental 4DVar (1)

\[
J(x) = \frac{1}{2}(x_b - x)^T P_b^{-1}(x_b - x) + \frac{1}{2} \sum_k (y_k - G_k(x))^T R_k^{-1}(y_k - G_k(x))
\]

\[
-\nabla J(x) = P_b^{-1}(x_b - x) + \sum_k G_k^T R_k^{-1}(y_k - G_k(x))
\]

where

\[
G_k = H_k M_{t_0 \rightarrow t_k}
\]

and

\[
G_k^T = M_{t_0 \rightarrow t_k}^T H_k^T
\]
Recall the linearisation state $x^{(m)}$ such that

$$x = x^{(m)} + \delta x^{(m)}$$

Then the cost function becomes

$$J(\delta x^{(m)}) = \frac{1}{2} (x_b - x^{(m)} - \delta x^{(m)})^T P_b^{-1} (x_b - x^{(m)} - \delta x^{(m)})$$
$$+ \frac{1}{2} (y - G (x_b - x^{(m)} - \delta x^{(m)}))^T R^{-1} (y - G (x_b - x^{(m)} - \delta x^{(m)}))$$
Recall the linearisation state $x^{(m)}$ such that

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Then the cost function becomes

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$$+ \frac{1}{2} (y - G(x_b - x^{(m)} - \delta x^{(m)}))^T R^{-1} (y - G(x_b - x^{(m)} - \delta x^{(m)}))$$

$$= \frac{1}{2} (x_b - x^{(m)} - \delta x^{(m)})^T P_b^{-1} (x_b - x^{(m)} - \delta x^{(m)})$$

$$+ \frac{1}{2} (d^{(m)} - G\delta x^{(m)})^T R^{-1} (d^{(m)} - G\delta x^{(m)})$$

where $d^{(m)} = y - G(x^{(m)})$
Weakly coupled assimilation in incremental 4DVar (3)

- \( \mathbf{d}^{(m)} = \mathbf{y} - \mathcal{G}(\mathbf{x}^{(m)}) \)
  - \( \mathcal{G}(\mathbf{x}^{(m)}) = \mathcal{G}\left(\begin{pmatrix} \mathbf{x}_1^{(m)} \\ \mathbf{x}_2^{(m)} \end{pmatrix}\right) \) is computed with the coupled nonlinear model

- \( G\delta \mathbf{x}^{(m)} = \begin{pmatrix} H_1 M_1 \delta \mathbf{x}_1^{(m)} \\ H_2 M_2 \delta \mathbf{x}_2^{(m)} \end{pmatrix} \)
  - Computed using the uncoupled linearised model and observation operator (suitably interpolated)

Thus in weakly coupled 4DVar the interaction between components happens though \( \mathcal{G} \) each outer loop of the minimisation
Many sequential techniques do not require the adjoint of the coupled model (i.e. Kalman filter, EnKF, particle filters, 3DVar, 4D-EnVar)

Thus the issue of multiple timescales are avoided

Similarly, the issue of an incomplete gradient is avoided
Coupled assimilation with sequential DA techniques

- Many sequential techniques do not require the adjoint of the coupled model (i.e. Kalman filter, EnKF, particle filters, 3DVar, 4DEnVar)
- Thus the issue of multiple timescales are avoided
- Similarly, the issue of an incomplete gradient is avoided
- Explicit cross-covariances may need to be specified (3DVar, particle filters)
- Localisation methods across the different components need to be specified (EnKF, 4DEnVar)
We have seen that in weakly coupled 4DVar, the only interaction between components comes through the evolution of the nonlinear coupled models.

- This concept can be extended to assimilation methods other than 4DVar.
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- The weak coupling refers to the coupling in the nonlinear trajectory.
More general weakly coupled assimilation

We have seen that in weakly coupled 4DVar, the only interaction between components comes through the evolution of the nonlinear coupled models.

- This concept can be extended to assimilation methods other than 4DVar.
- The weak coupling refers to the coupling in the nonlinear trajectory.
- Each component does not need to use 4DVar as its assimilation method.
We have seen that in weakly coupled 4DVar, the only interaction between components comes through the evolution of the nonlinear coupled models.

- This concept can be extended to assimilation methods other than 4DVar.
- The weak coupling refers to the coupling in the nonlinear trajectory.
- Each component does not need to use 4DVar as its assimilation method.
- For example, one could use a (simplified extended) Kalman filter for a soil moisture model.
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Potential benefits of coupled assimilation

Coupled assimilation at ECMWF
Observations of one component of the system can update another, e.g.

- Observing surface winds could directly impact on the land surface model, e.g. through updating both surface temperatures and evaporation rates
- Observations of passive (chemical) tracers can be used to update atmospheric winds
Temperature errors are correlated
Atmosphere Temperature - Ocean Temperature

Wind speeds affect ocean cooling rates
Atmosphere Zonal Wind - Ocean Temperature

Coupled balances

Ekman circulation
Atmosphere Zonal Wind - Ocean Meridional Current

Reduced initialisation shock

Uncoupled assimilation

Coupled assimilation

Balanced ocean-atmosphere analysis

Global net air-sea fluxes toward the ocean in CERA-20C and ORA-20C.

▶ Spurious trend in ORA-20C probably due to shift in wind forcing in ERA-20C (heat lost)

Ocean temperature increment in CERA-20C and ORA-20C.

▶ Increment in ORA-20C is trying to compensate for the heat lost
▶ CERA-20C fluctuates around zero suggesting a more balanced air-sea interface

Courtesy of E. de Boisséson
Cyclone tracking

Ocean temperature at 40 metres observed by an Argo float located on the track of the cyclone Phailin, 11 October 2013 in the Bay of Bengal.

The temperature drop is due to the cold wake induced by the cyclone. The difference between the red and the black thick lines shows the impact of using a coupled assimilation system. Improvement through the better use of surface wind satellite measurements.

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Coupled assimilation at ECMWF
"ECMWF has started to explore a new coupled assimilation system to initialise the numerical weather forecast in a more comprehensive and balanced manner. Such an approach has the potential to better use satellite measurements and to improve the quality of our forecasts. It will generate a reduction of initialisation shocks in coupled forecasts by fully accounting for interactions between the components. It will also lead to the generation of a consistent Earth-system state for the initialisation of forecasts across all timescales",
ECMWF Roadmap to 2025
Components of the Earth System
Coupled Models

- Atmosphere
- Sea Ice
- Ocean
- Land
- Waves

- HRES NWP and ERA5

Training course 2017 - Coupled DA
Components of the Earth System

Coupled Models

- Atmosphere
- Sea Ice
- Ocean
- Land
- Waves

- ENS/monthly, seasonal, and CERA

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Components of the Earth System
Coupled Assimilation

- Atmosphere
  - 4D-Var
- Sea Ice
  - 3D-Var
- Ocean
  - 3D-Var
- Waves
  - OI
- Land
  - EKF

- HRES NWP and ERA5: land and waves weakly coupled
Components of the Earth System
Coupled Assimilation

- Atmosphere 4D-Var
- Sea Ice 3D-Var
- Ocean 3D-Var
- Waves OI
- Land EKF

ENZ, monthly and seasonal, Ocean5/ORAS5: uncoupled ocean and sea ice assimilation
Components of the Earth System
Coupled Assimilation

- Atmosphere 4D-Var
- Sea Ice 3D-Var
- Ocean 3D-Var
- Waves OI
- Land EKF

► CERA-20C: outer-loop coupling for atm-ocean, sea ice

Hence different coupling strategies are used for the different configurations

ECMWF
EUROPEAN CENTRE FOR MEDIUM-RANGE WEATHER FORECASTS

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Components of the Earth System

Coupled Assimilation

- **Atmosphere**
  - 4D-Var

- **Sea Ice**
  - 3D-Var

- **Ocean**
  - 3D-Var

- **Waves**
  - OI

- **Land**
  - EKF

- **CERA-20C**: outer-loop coupling for atm-ocean, sea ice
- **CERA-SAT**: with also land-atm weak coupling and full observing system

EUMWF

European Centre for Medium-Range Weather Forecasts

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Components of the Earth System

Coupled Assimilation

- Atmosphere 4D-Var
- Sea Ice 3D-Var
- Ocean 3D-Var
- Waves OI
- Land EKF

- CERA-20C: outer-loop coupling for atm-ocean, sea ice
- CERA-SAT with also land-atm weak coupling and full observing system
- Hence different coupling strategies are used for the different configurations
Consistency of the coupling approaches across the different components of the Earth system

Comprehensive Earth system approach; atmosphere, land, ocean, sea ice, waves
Towards an Earth System Approach

Coupled Assimilation - CERAv3/CERA100

- Consistency of the coupling approaches across the different components of the Earth system
- Comprehensive Earth system approach; atmosphere, land, ocean, sea ice, waves, atmospheric composition
Summary

- Coupled data assimilation is, in theory, the same as multivariate DA
- Coupled data assimilation can improve balance in analyses and can increase the use of, and information gained from, observations
- Issues arise from:
  - Varying timescales in the different components - leads to poor TL approximation for "long" windows
  - Various components of the Earth system running separate models/executables - the full adjoint is not always available
- Weakly coupled assimilation is coupling at the outer loop level, where the full nonlinear model is coupled
- ECMWF is regularly doing coupled assimilation:
  - Atmosphere - land - wave in high resolution NWP and ERA5 reanalysis
  - Atmosphere - land - wave - sea ice - ocean in CERA-20C and CERA-SAT reanalyses
- Specifying cross-covariances is a big future challenge